Post-Quantum

Cryptography Conference

LMS: Lighter, faster key generation

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LMS: Faster key generation, lighter keys

Francisco José Vial-Prado

Post-Quantum Cryptography Conference

7-8 November, 2023, Amsterdam (NL)

Fortanix

This talk

- **Introduction to LMS** *(Also see Volker Krummel's talk before lunch "Stateful Hash-Based Signature Schemes")*
- **Faster key generation** (Remarks on SIMD versions of RFC8554 algorithms)
- **Key size/signature speed trade-offs** (Recalling the "treehash" algorithms)

This talk

- **Introduction to LMS** *(Also see Volker Krummel's talk before lunch "Stateful Hash-Based Signature Schemes")*
- **Faster key generation** (Remarks on SIMD versions of RFC8554 algorithms)
- **Key size/signature speed trade-offs** (Recalling the "treehash" algorithms)

NOT This talk

State management, interoperability, export restrictions …

LMS = LM-OTS/LMS/HSS...

LMS is a *stateful hash-based signature scheme*

- Key generation requires hashing
- Signing a message requires hashing
- Verifying a signature requires hashing

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Also...

- There is an internal state that MUST evolve upon signing (typically, one counter).
- LMS keys can be organized into HSS keys, augmenting capacity

LMS = LM-OTS/LMS/HSS…

$$\boxed{\text{LM-OTS key}} \xrightarrow{\times 2^{h} \ \ (\leq 2^{25})} \boxed{\text{LMS key}} \xrightarrow{\times k \ \ (\leq 8)} \boxed{\text{HSS key}}$$

LMS = LM-OTS/LMS/HSS...

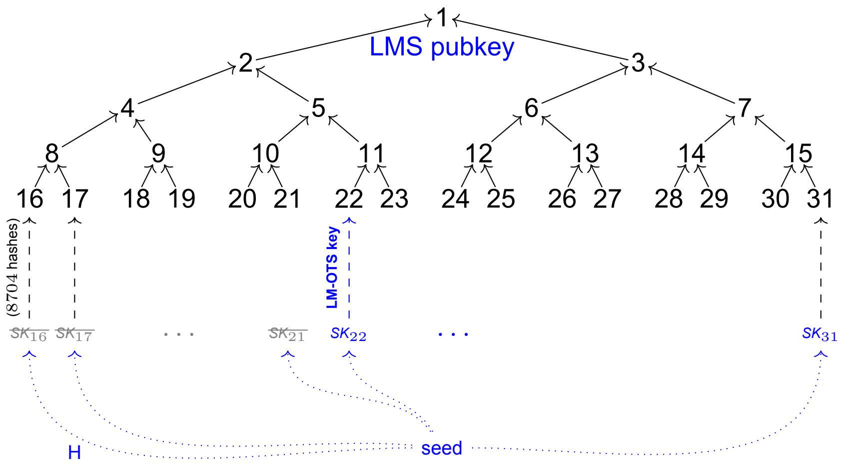
$$\begin{tikzcd} \boxed{\text{LM-OTS key}} \arrow[r, "\times 2^{h} \ (\leq 2^{25})"] & \boxed{\text{LMS key}} \arrow[r, "\times k \ (\leq 8)"] & \boxed{\text{HSS key}} \\ \text{(1 sig)} & (2^{h} \text{ sigs}) & (2^{h_1+\cdots+h_k} \text{ sigs}) \end{tikzcd}$$

LMS = LM-OTS/LMS/HSS...

$$\boxed{\text{LM-OTS key}} \xrightarrow{\times 2^{h}\ \ (\leq 2^{25})} \boxed{\text{LMS key}} \xrightarrow{\times k\ \ (\leq 8)} \boxed{\text{HSS key}}$$

(1 sig)	(2^{h} sigs)	($2^{h_1+\cdots+h_k}$ sigs)
Keygen:		
$34 \times 255 + 34$ hashes	2^{h+1} hashes	$k-1$ sigs
(use once)	(maintain state)	(rotate exhausted LMS keys)

LMS = LM-OTS/LMS/HSS...



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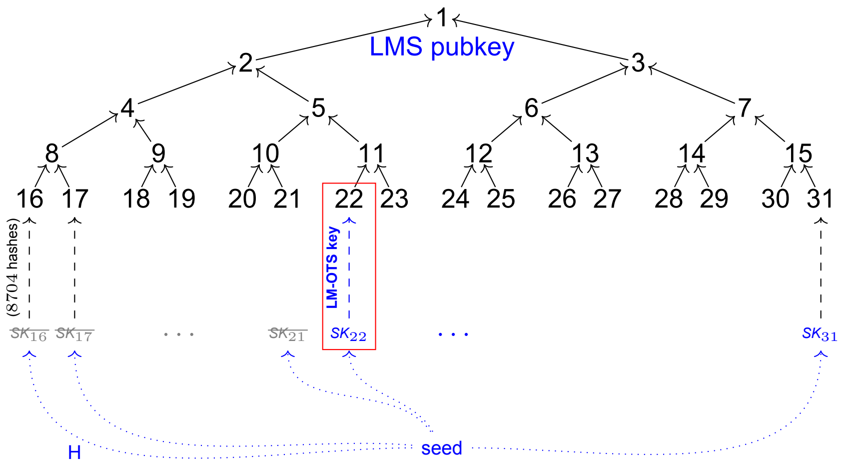


Single leaf calculation

$SK_{22} \xrightarrow{\textbf{LM-OTS key}} 22$

Single leaf calculation

$$
\begin{tikzcd}
SK_{22} \arrow[rrrr, dashed, "\textbf{LM-OTS key}"] \arrow[d, dotted] \arrow[dd, dotted, bend right] \arrow[dddd, dotted, bend right] & & & & 22 \\
x[0] \arrow[r] & h^1(x[0]) \arrow[r] & h^2(x[0]) \arrow[r] & \cdots \arrow[r] & h^{255}(x[0]) \arrow[d, dotted] \\
x[1] \arrow[r] & h^1(x[1]) \arrow[r] & h^2(x[1]) \arrow[r] & \cdots \arrow[r] & h^{255}(x[1]) \arrow[d, dotted] \\
\vdots & & & & \vdots \arrow[d, dotted] \\
x[33] \arrow[r] & h^1(x[33]) \arrow[r] & h^2(x[33]) \arrow[r] & \cdots \arrow[r] & h^{255}(x[33]) \arrow[uuuu, dotted, bend right]
\end{tikzcd}
$$

SHA-256 in SIMD is Easy™

- SHA-256 operates on 32-bit words
- Only uses bit shifts, rotation, and wrapping addition

SHA-256 in SIMD is Easy™

- SHA-256 operates on 32-bit words
- Only uses bit shifts, rotation, and wrapping addition
- Can compute `LANES` hash values In One Go!

Single leaf calculation

$$SK_{22} \dashrightarrow{} 22 \quad \textbf{LM-OTS key}$$

$$\begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \end{bmatrix} \xrightarrow{\text{SIMD}_4} \begin{bmatrix} h^1(x[0]) \\ h^1(x[1]) \\ h^1(x[2]) \\ h^1(x[3]) \end{bmatrix} \longrightarrow \begin{bmatrix} h^2(x[0]) \\ h^2(x[1]) \\ h^2(x[2]) \\ h^2(x[3]) \end{bmatrix} \longrightarrow \cdots \longrightarrow \begin{bmatrix} h^{255}(x[0]) \\ h^{255}(x[1]) \\ h^{255}(x[2]) \\ h^{255}(x[3]) \end{bmatrix}$$

Single leaf calculation

$$\dashrightarrow SK_{22} \xrightarrow{\textbf{LM-OTS key}} 22$$

$$\begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \end{bmatrix} \xrightarrow{\text{SIMD}_4} \begin{bmatrix} h^1(x[0]) \\ h^1(x[1]) \\ h^1(x[2]) \\ h^1(x[3]) \end{bmatrix} \longrightarrow \begin{bmatrix} h^2(x[0]) \\ h^2(x[1]) \\ h^2(x[2]) \\ h^2(x[3]) \end{bmatrix} \longrightarrow \cdots \longrightarrow \begin{bmatrix} h^{255}(x[0]) \\ h^{255}(x[1]) \\ h^{255}(x[2]) \\ h^{255}(x[3]) \end{bmatrix}$$

$$\Rightarrow \left\lceil \frac{34}{\mathtt{LANES} \times \mathtt{THREADS}} \right\rceil \times 255 + 34 \text{ calls}$$

LM-OTS signing

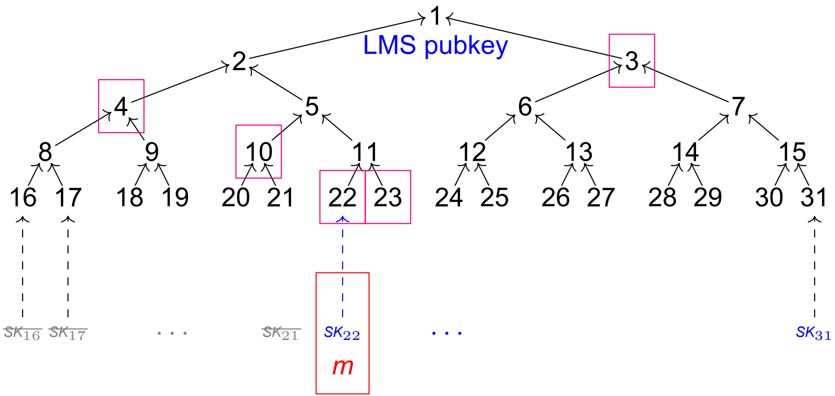
$$\begin{tikzcd}
SK_{22} \arrow[rrrrr, dashed, "\textbf{LM-OTS key}"] & & & & & 22 \\
& & \boxed{h^2(x[0])} \arrow[r] & \cdots \arrow[r] & h^{255}(x[0]) \arrow[d, dotted] & \\
& \boxed{h^1(x[1])} \arrow[r] & h^2(x[1]) \arrow[r] & \cdots \arrow[r] & h^{255}(x[1]) \arrow[d, dotted] & \\
\vdots & & & & \vdots \arrow[d, dotted] & \\
& & \boxed{h^{42}(x[33])} \arrow[r] & \cdots \arrow[r] & h^{255}(x[33]) \arrow[uuuur, dotted, bend right] &
\end{tikzcd}$$

Signer reveals intermediate values

Verifier hashes again

(Message dependency ends here)

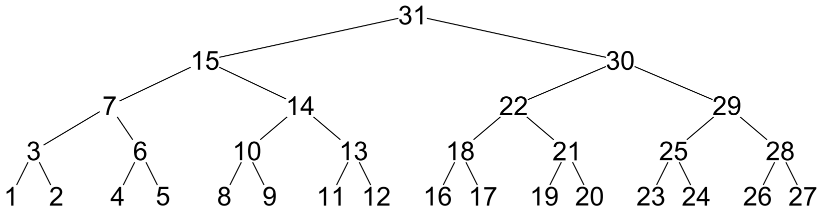
LMS signing



Signer needs to provide $\{23, 10, 4, 3\}$

Verifier hashes again

LMS root (RFC8554 app. C)



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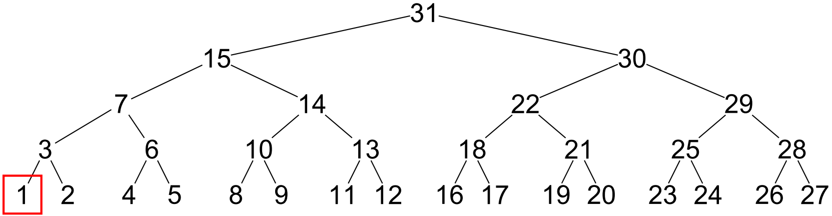

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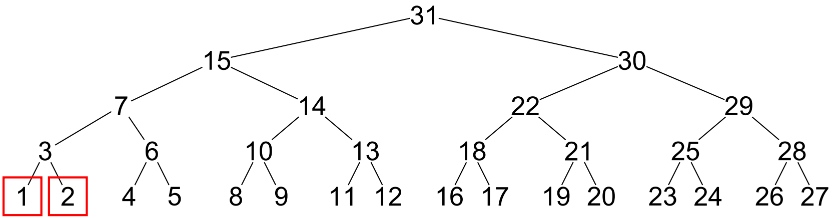

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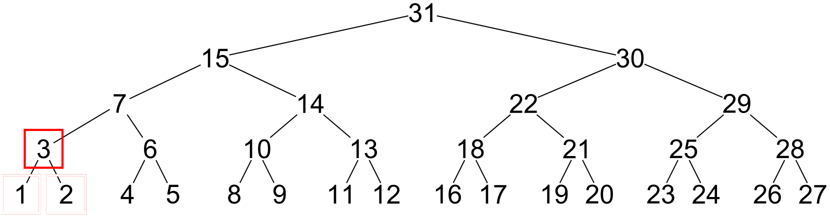

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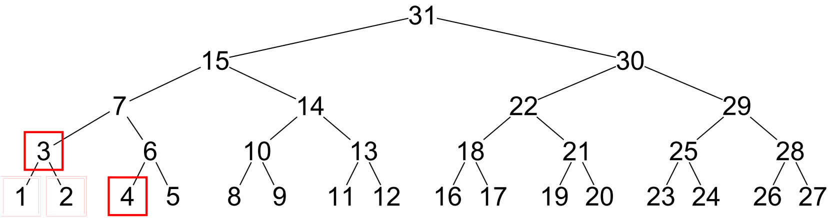

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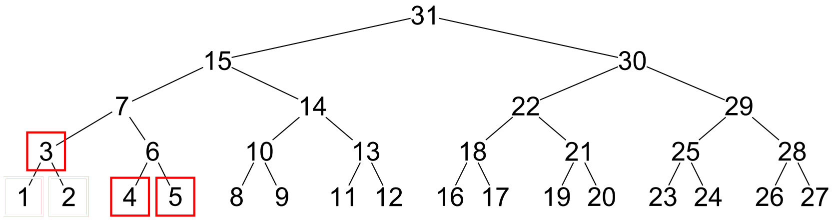

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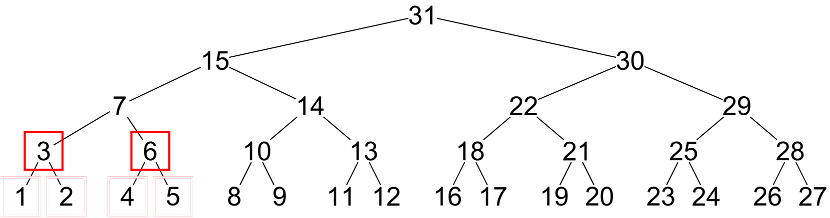

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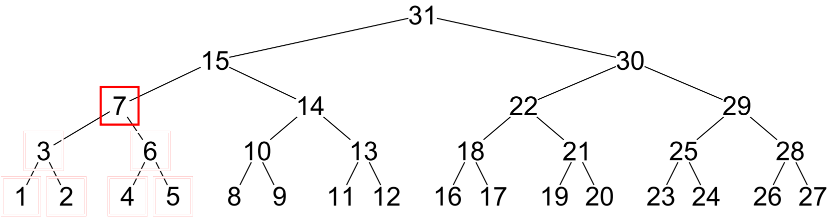

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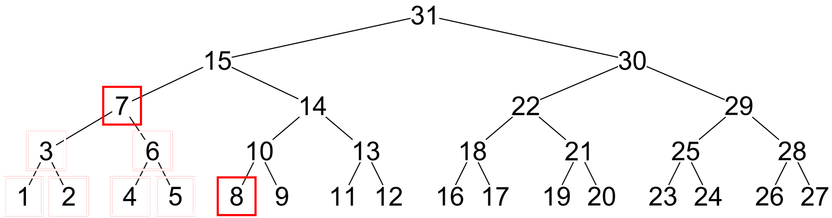

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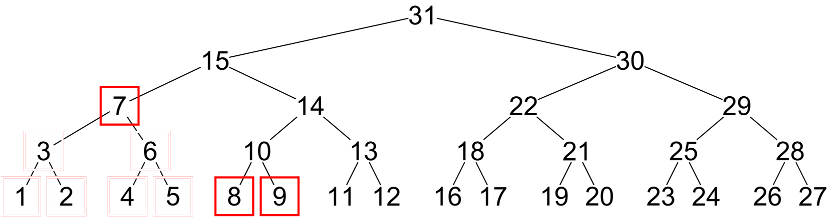

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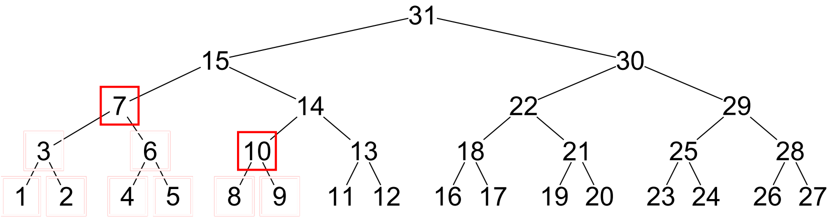

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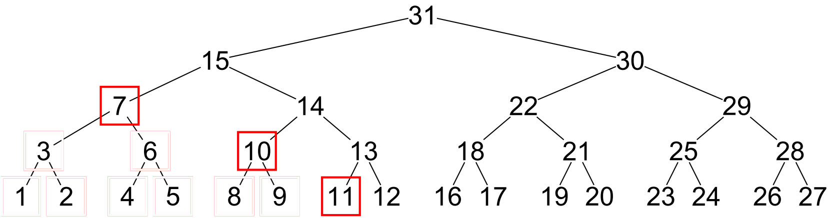

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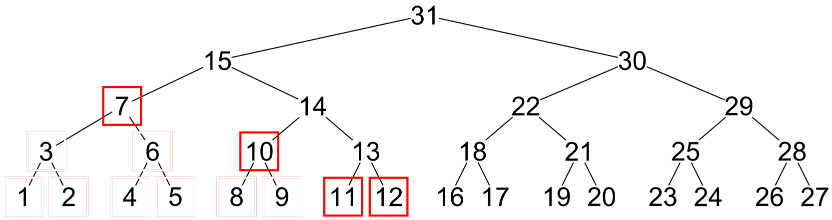


Get to the root with a stack of h -1 hashes! (≤ 768 bytes)

LMS root (RFC8554 app. C)

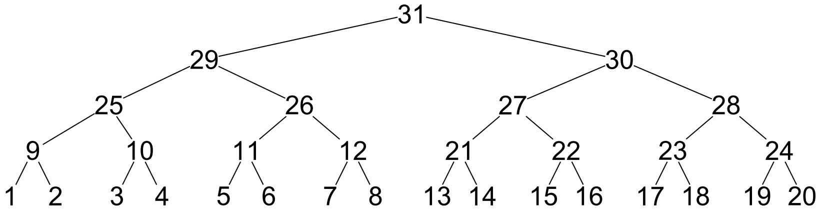
```
// Generating an LMS Public Key from an LMS Private Key

for ( i = 0; i < 2^h; i = i + 1 ) {
  r = i + 2^h;
  temp = H(I || r || "D_LEAF" || OTS_PUB_HASH[i])  // Compute leaf
  j = i;
  while (j % 2 == 1) {
    r = (r - 1) / 2;
    j = (j - 1) / 2;
    left = pop(data stack);
    temp = H(I || r || "D_INTR" || left || temp)  // Compute branch
  }
  push temp onto the data stack
}
public_key = pop(data stack)
```

SIMD LMS root (`LANES = 4`)

Stack = vector of arrays of `LANES` nodes.

As soon as $2 \times$ `LANES` neighbour nodes are available, hash them into `LANES` nodes.



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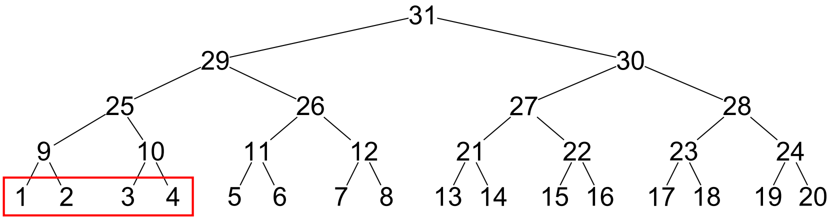

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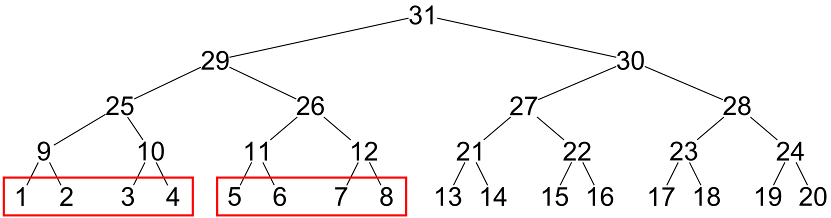

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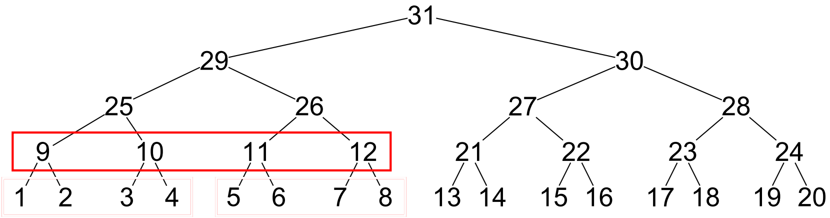

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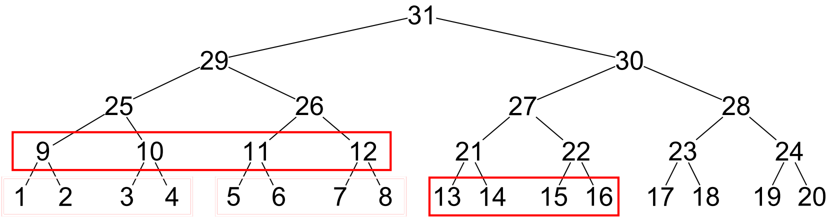

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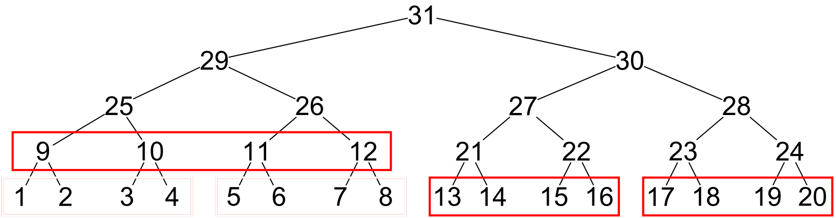

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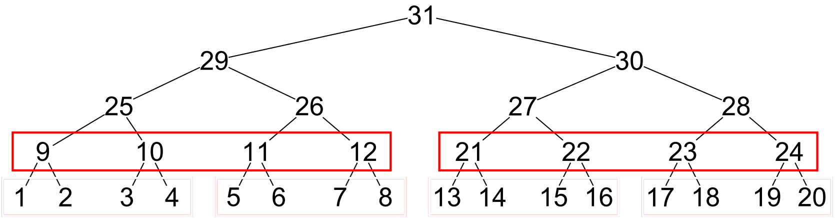

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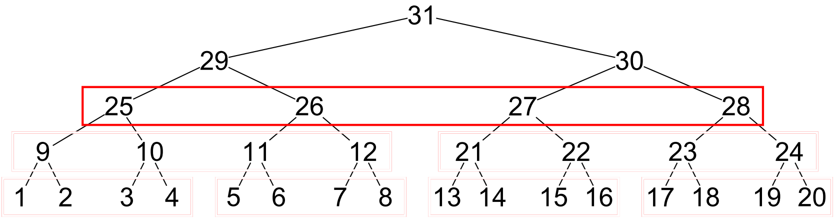

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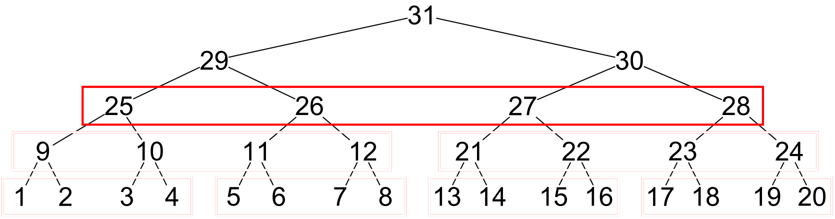

Get to level log($\mathtt{LANES}$) with a stack of $(h-1) \times$ LANES SIMD calls

```
for ( i = 0; i < 2^h; i = i + LANES ) {
  r = i + 2^h;
  temp = H(
    I || r + 0..LANES || "D_LEAF" || OTS_PUB_HASH[0..LANES]
  )
  j = i / LANES;
  while (j % 2 == 1) {
    r = (r - LANES) / 2;
    j = (j - LANES) / 2;
    left = pop(data stack);
    temp = H(
      I || r + [0..LANES] || "D_INTR" || left || temp[0..LANES]
    )
  }
  push temp onto the data stack
}
// Compute levels [0..log(LANES)]
```

SIMD LMS KeyGen

$$\boxed{\text{LM-OTS key}} \xrightarrow{\times 2^{h} \ \ (\leq 2^{25})} \boxed{\text{LMS key}} \xrightarrow{\times k \ \ (\leq 8)} \boxed{\text{HSS key}}$$

SIMD LMS KeyGen

$$
\begin{tikzcd}
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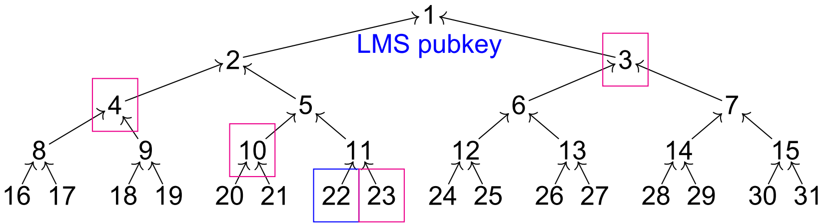
\end{tikzcd}
$$

$$\left\lceil \frac{34}{\mathtt{LANES}\cdot\mathtt{THREADS}} \right\rceil \cdot 255 + 34 \qquad \left\lceil \frac{2^{h+1}}{\mathtt{LANES}\cdot\mathtt{THREADS}} \right\rceil \qquad \left\lceil \frac{k-1}{\mathtt{THREADS}} \right\rceil \text{ sigs}$$


1
LMS pubkey
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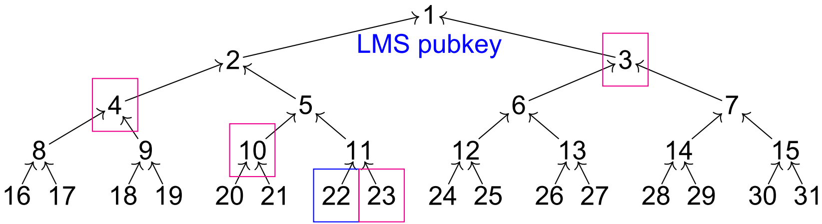
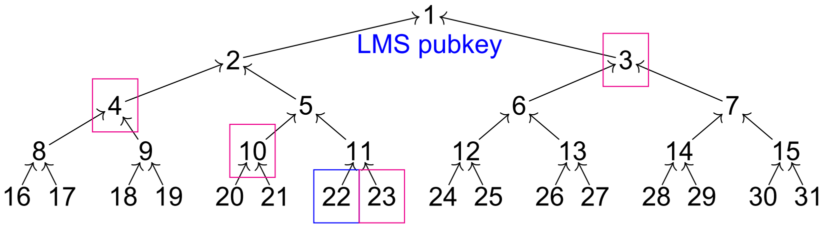


Light, slow

Remember the state and seed

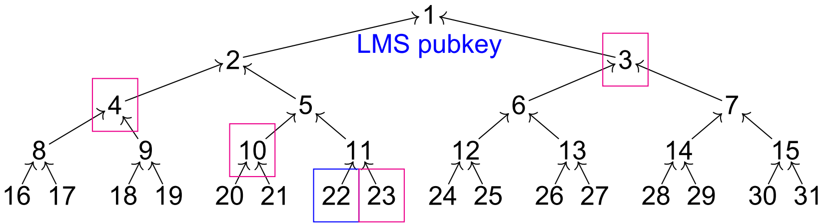


Light, slow

Remember the state and seed

Heavy, fast

Remember **everything**



Light, slow

Remember the state and seed

Heavy, fast

Remember **everything**

`Everything` **=** $2^{h+1} \times 32$ **bytes (**≤ 2.14 **GB)**

Node lifetime

$$
\begin{aligned}
\texttt{life}(h, 0) &= [0, 2) & \texttt{(leftmost leaf)} \\
\texttt{life}(h, 2) &= [2, 4) \\
\texttt{life}(h-1, 0) &= [0, 4) \\
\texttt{life}(1, 0) &= [0, 2^h) & \texttt{(left child of root)} \\
\texttt{life}(1, 1) &= [0, 2^h) & \texttt{(right child of root)} \\
&\vdots
\end{aligned}
$$

Node lifetime

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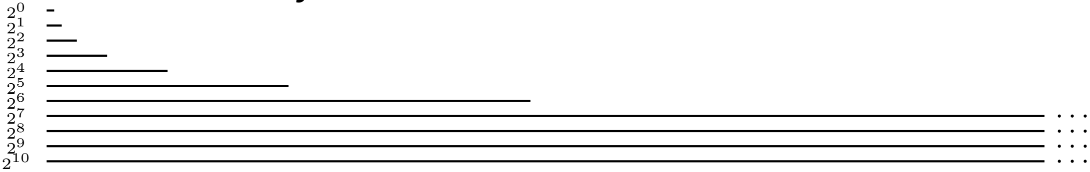
\texttt{life}(1,1) &= [0,2^h) && \texttt{(right child of root)} \\
&\vdots
\end{aligned}
$$

$$
\boxed{\texttt{life}(l,i) = \left[2^{h-l+1}\left\lfloor i/2 \right\rfloor, 2^{h-l+1}\left\lceil (i+1)/2 \right\rceil\right)}
$$

At level $l \in \{1, \ldots, h\}$, node $i \in \{0, \ldots, 2^l - 1\}$ lives during $2^{h-l+1} - 1$ signatures

Small-Memory LM Schemes



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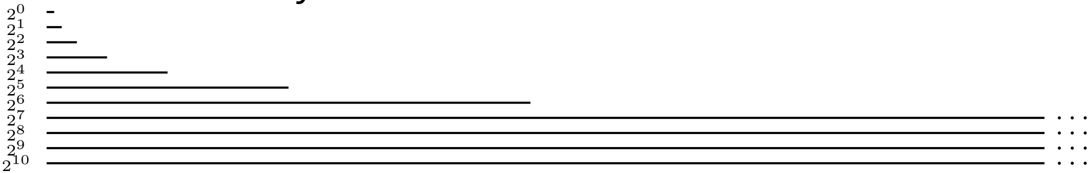
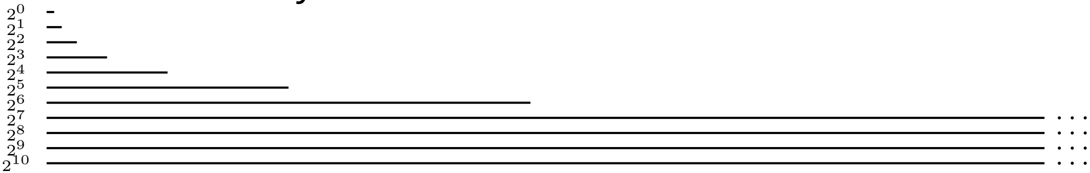


$N^{1/2}$ algorithm

Remember top $h/2$ levels entirely. On level $l > h/2$, remember nodes $\{0, \ldots, 2^{l-h/2}\}$. Remember leaves $\{0, \ldots, 2^{h/2} + h/2\}$.

Small-Memory LM Schemes



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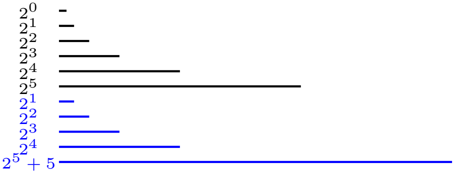
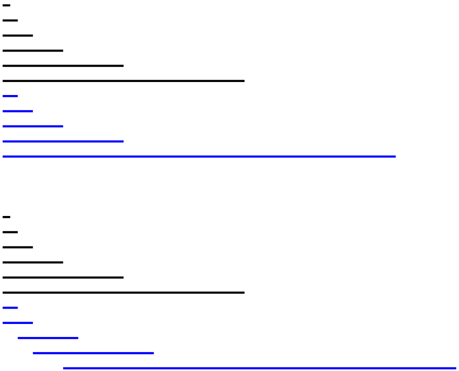


Small-Memory LM Schemes

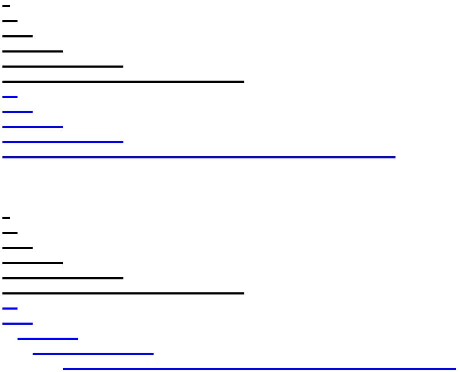
Slide windows after signing



At signature k, compute one leaf and upper branches as possible. Forget leaves left of $L := k - 2^{h/2} - h/2$ and nodes left of $L/2^{h-l}$.

Small-Memory LM Schemes

Slide windows after signing



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State

"State" = counter $+$ cached nodes.

2.14 GB $\rightarrow$ **1 MB** for $h = 25$ and SHA-256

All together!

SHA-256 in SIMD is easy!

KeyGen

- Use SIMD/multithreading to compute leaves
- Use SIMD/multithreading to get to the root faster
- Remember node windows according to $N^{1/2}$ algorithm

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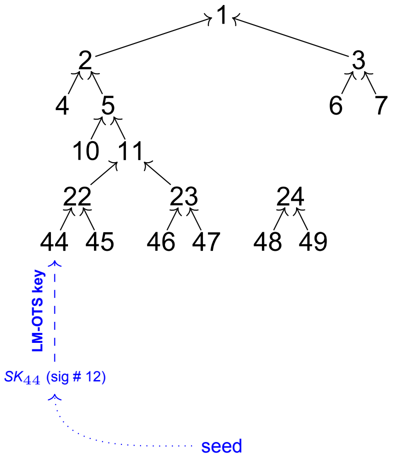
- Remember node windows according to $N^{1/2}$ algorithm

Sign

- Use SIMD/multithreading to compute one leaf ($\lceil 34/(L \cdot T) \rceil \cdot 255 + 34$ calls)
- Compute at most $h/2$ branches
- Forget nodes left nodes past their lifetime
- Release signature AFTER

Thank you!


1
2
3
4
5
6
7
10
11
22
23
24
44
45
46
47
48
49
LM-OTS key
SK_{44} (sig # 12)
seed

LMS: Faster key generation, lighter keys

Francisco José Vial-Prado

Post-Quantum Cryptography Conference

7-8 November, 2023, Amsterdam (NL)

Fortanix

Post-Quantum Cryptography Conference

PKI Consortium